AER E 351 Homework 03

Matthew Mehrtens

# Problem 1

% AER E 351 Homework 03 Problem 1a

% Matthew Mehrtens

clear, clc, close all;

%% Given

R\_Earth = 6.37812e3; % [km]

mu\_Earth = 3.986e5; % [km^3/s^2]

a = 1.7 \* R\_Earth; % [km]

e = 0.4; % []

i = deg2rad(20); % [rad]

Omega = deg2rad(30); % [rad]

omega = deg2rad(10); % [rad]

f\_0 = deg2rad(60); % [rad]

theta = omega + f\_0; % [rad]

%% Calculations

r\_mag = a \* (1 - e^2) / (1 + e \* cos(f\_0)); % [km]

r = r\_mag \* [...

cos(Omega) \* cos(theta) - sin(Omega) \* sin(theta) \* cos(i)

sin(Omega) \* cos(theta) + cos(Omega) \* sin(theta) \* cos(i)

sin(theta) \* sin(i)]; % [km]

h = sqrt(mu\_Earth \* a \* (1 - e^2)); % [km^2/s]

v = mu\_Earth / h \* [...

-(cos(Omega) \* (sin(theta) + e \* sin(omega)) + sin(Omega) \* (cos(theta) + e \* cos(omega)) \* cos(i))

-(sin(Omega) \* (sin(theta) + e \* sin(omega)) - cos(Omega) \* (cos(theta) + e \* cos(omega)) \* cos(i))

(cos(theta) + e \* cos(omega)) \* sin(i)]; % [km/s]

%% Display

fprintf(...

"|r| = %g km\n" + ...

"r [km]:\n", ...

r\_mag);

disp(r);

fprintf(...

"h = %g km^2/s\n" + ...

"v [km/s]:\n", ...

h);

disp(v);

% AER E 351 Homework 03 Problem 1b

% Matthew Mehrtens

clear, clc, close all;

%% Given

R\_Earth = 6.37812e3; % [km]

mu\_Earth = 3.986e5; % [km^3/s^2]

a = 2 \* R\_Earth; % [km]

e = 0; % []

i = deg2rad(10); % [rad]

Omega = deg2rad(30); % [rad]

theta\_0 = deg2rad(20); % [rad]

%% Calculations

r\_mag = a; % [km]

r = r\_mag \* [...

cos(Omega) \* cos(theta\_0) - sin(Omega) \* sin(theta\_0) \* cos(i)

sin(Omega) \* cos(theta\_0) + cos(Omega) \* sin(theta\_0) \* cos(i)

sin(theta\_0) \* sin(i)]; % [km]

v\_mag = sqrt(mu\_Earth \* (2 / r\_mag - 1 / a)); % [km/s]

h = sqrt(mu\_Earth \* a \* (1 - e^2)); % [km^2/s]

gamma = acos(h / (r\_mag \* v\_mag)); % [rad]

c = cross(r, [cos(Omega) sin(Omega) 0]); % [km]

syms v\_x v\_y v\_z;

S = solve(...

v\_mag^2 == v\_x^2 + v\_y^2 + v\_z^2, ...

r\_mag \* v\_mag \* sin(gamma) == r(1) \* v\_x + r(2) \* v\_y + r(3) \* v\_z, ...

0 == c(1) \* v\_x + c(2) \* v\_y + c(3) \* v\_z, ...

v\_x, v\_y, v\_z);

v = [double(S.v\_x(2)) double(S.v\_y(2)) double(S.v\_z(2))]; % [km/s]

% v = mu\_Earth / h \* [...

% -(cos(Omega) \* (sin(theta) + e \* sin(omega)) + sin(Omega) \* (cos(theta) + e \* cos(omega)) \* cos(i))

% -(sin(Omega) \* (sin(theta) + e \* sin(omega)) - cos(Omega) \* (cos(theta) + e \* cos(omega)) \* cos(i))

% (cos(theta) + e \* cos(omega)) \* sin(i)]; % [km/s]

%% Display

fprintf(...

"|r| = %g km\n" + ...

"r [km]:\n", ...

r\_mag);

disp(r);

fprintf(...

"|v| = %g km/s\n" + ...

"h = %g km^2/s\n" + ...

"gamma = %g rad\n" + ...

"c [km]:\n", ...

v\_mag, h, gamma);

disp(c);

fprintf("v [km/s]:\n");

disp(v);

# Problem 3.2

% AER E 351 Homework 03 Problem 3.2a

% Matthew Mehrtens

clear, clc, close all;

%% Given

r = [-3 \* sqrt(2) / 2

0

-3 \* sqrt(2) / 2];

v = [0

-sqrt(6) / 6

0];

%% Calculations

r\_mag = norm(r);

v\_mag = norm(v);

% a = r / (2 - rv^2 / mu)

a = r\_mag / (2 - r\_mag \* v\_mag^2);

% e = [(|v|^2 - mu / |r|) \* r - (r . v) \* v]

% dot(r, v) % <-- r.v = 0

e = (v\_mag^2 - 1 / r\_mag) \* r;

e\_mag = norm(e);

h = cross(r, v);

h\_mag = norm(h);

i = acosd(h(3) / h\_mag); % [°]

n = cross([0 0 1], h / h\_mag);

n\_mag = norm(n);

% Since n\_y is < 0, we need a correction on omega

Omega = 180 + acosd(n(1) / n\_mag);

omega = acosd(dot(n, e) / (n\_mag \* e\_mag)); % [°]

f = acosd(dot(e, r) / (e\_mag \* r\_mag)); % [°]

%% Output

fprintf( ...

"Problem 3.2a Solutions:\n" + ...

"a = %g\n" + ...

"|e| = %g\n" + ...

"i = %g°\n" + ...

"n\_y = %g\n" + ...

"Omega = %g°\n" + ...

"omega = %g°\n" + ...

"f = %g°\n", ...

a, e\_mag, i, n(2), Omega, omega, f);

% AER E 351 Homework 03 Problem 3.2a

% Matthew Mehrtens

clear, clc, close all;

%% Given

r = [sqrt(2) / 2

sqrt(2) / 2

0];

v = [-sqrt(2) / 2

sqrt(2) / 2

0];

%% Calculations

r\_mag = norm(r);

v\_mag = norm(v);

% a = r / (2 - rv^2 / mu)

a = r\_mag / (2 - r\_mag \* v\_mag^2);

% e = [(|v|^2 - mu / |r|) \* r - (r . v) \* v]

% dot(r, v) % <-- r.v = 0

e = (v\_mag^2 - 1 / r\_mag) \* r;

e\_mag = norm(e);

h = cross(r, v);

h\_mag = norm(h);

i = acosd(h(3) / h\_mag); % [°]

n = cross([0 0 1], h / h\_mag);

n\_mag = norm(n);

% Since n\_y is < 0, we need a correction on omega

Omega = 180 + acosd(n(1) / n\_mag);

omega = acosd(dot(n, e) / (n\_mag \* e\_mag)); % [°]

f = acosd(0 / 0); % [°]

%% Output

fprintf( ...

"Problem 3.2b Solutions:\n" + ...

"a = %g\n" + ...

"|e| = %g\n" + ...

"i = %g°\n" + ...

"Omega = %g°\n" + ...

"omega = %g°\n" + ...

"f = %g°\n", ...

a, e\_mag, i, Omega, omega, f);

# Problem 3.9

Desmos Graph: <https://www.desmos.com/calculator/akqqfmk1ib>

% AER E 351 Homework 03 Problem 3.9

% Matthew Mehrtens

clear, clc, close all;

%% Given

e = 0.5; % []

f = 45; % [°]

%% Problem 3.9a

A graph of a circle with a circle in the center

Description automatically generated

gamma\_a = atand(0.707 / 0.707) - atand((0.707 - 0.293) / 0.707); % [°]

verify\_gamma\_a = atand(e \* sind(f) / (1 + e \* cosd(f))); % [°]

%% Problem 3.9b

A graph of a circle with a circle in the center

Description automatically generated

gamma\_min = -asind(e); % [°]

%% Problem 3.9c

A graph of a circle with a circle and a line in the center

Description automatically generated

p\_coeff\_c = 1 / (1 + 0.354); % []

verify\_p\_coeff\_c = 1 / (1 + e \* cosd(f)); % []

%% Problem 3.9d

A graph of a circle with a number in the center

Description automatically generated

p\_coeff\_d = 1 / (1 - 0.25); % []

verify\_p\_coeff\_d = 1 / (1 + e \* cosd(240)); % []

%% Output

fprintf( ...

"Problem 3.9 Solutions:\n" + ...

"a:\n" + ...

"gamma = %g°\n" + ...

"verification: gamma = %g°\n" + ...

"b:\n" + ...

"gamma\_min = %g°\n" + ...

"c:\n" + ...

"r = %gp\n" + ...

"verification: r = %gp\n" + ...

"d:\n" + ...

"r = %gp\n" + ...

"verification: r = %gp\n" + ...

"e:\n" + ...

"Yes, by using Equation 1.33:\n" + ...

"\tr = a \* (1 - e^2) / (1 + e \* cos(f))\t(Eq. 1.33)\n", ...

gamma\_a, verify\_gamma\_a, gamma\_min, p\_coeff\_c, verify\_p\_coeff\_c, ...

p\_coeff\_d, verify\_p\_coeff\_d);

# Problem 3.10

Desmos Graph: <https://www.desmos.com/calculator/j03kwfoowj>

% AER E 351 Homework 03 Problem 3.9

% Matthew Mehrtens

clear, clc, close all;

%% Given

e = 2; % []

%% Problem 3.10a

A graph paper with a circle and a red circle with a red triangle and a red triangle with a red line and a black triangle with a red line and a black triangle with a red line and

Description automatically generated

delta = 2 \* (90 - atand((e - 0.5) / 0.866)); % [°]

verify\_delta = 2 \* asind(1 / e); % [°]

%% Problem 3.10b

A graph paper with a circle and a red circle with a red triangle and a red triangle with a red line and a black triangle with a red line and a black triangle with a red line and

Description automatically generated

f\_inf = 90 + atand(0.5 / 0.866); % [°]

verify\_f\_inf = 180 - acosd(1 / e); % [°]

%% Problem 3.10c

A graph paper with a circle and a red circle with a red triangle and a red triangle with a red line and a black triangle with a red line and a black triangle with a red line and

Description automatically generated

v\_inf = sqrt(0.866^2 + (e - 0.5)^2); % [mu/h]

verify\_v\_inf = sqrt(e^2 - 1); % [mu/h]

%% Problem 3.10d

A graph with a circle and a triangle in the center

Description automatically generated

p\_coeff = 1 / (e - 0.268); % []

%% Problem 3.10e

A graph with a circle and a circle with a red circle and a black line

Description automatically generated

e\_1 = 1.414;

%% Output

fprintf( ...

"Problem 3.10 Solutions:\n" + ...

"a:\n" + ...

"delta = %g°\n" + ...

"verification: delta = %g°\n" + ...

"b:\n" + ...

"f\_inf = %g°\n" + ...

"verification: f\_inf = %g°\n" + ...

"c:\n" + ...

"v\_inf = %g mu/h\n" + ...

"verification: v\_inf = %g mu/h\n" + ...

"d:\n" + ...

"Delta = %gp\n" + ...

"e:\n" + ...

"e = %g\n", ...

delta, verify\_delta, f\_inf, verify\_f\_inf, v\_inf, verify\_v\_inf, ...

p\_coeff, e\_1);